

DUALITY AND FACTORIZATION THEOREM IN QCD¹

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Abstract

We find that in “two-photon”-like processes in the scalar φ_E^3 model and also in hadron-pair production arising from the collisions of a real (transversely polarized) and a highly virtual, longitudinally polarized, photon in QCD, there is duality between two distinct nonperturbative mechanisms. These two mechanisms, one involving a twist-3 Generalized Distribution Amplitude, the other employing a leading-twist Transition Distribution Amplitude, are associated with different regimes of factorization. In the kinematical region, where the two mechanisms overlap, duality is observed for the scalar φ_E^3 model, while in the QCD case the appearance of duality turns out to be sensitive to the particular nonperturbative model applied and can, therefore, be used as a tool for selecting the most appropriate one.

1 Introduction

The only known method today of applying QCD in a rigorous way is based on the factorization of the dynamics and the isolation of a short-distance part that becomes accessible to perturbative techniques of quantum field theory (see, [1, 2, 3] and for a review, for instance, [4] and references cited therein). Then, the conventional systematic way of dealing with the long-distance part is to parametrize it in terms of matrix elements of quark and gluon operators between hadronic states (or the vacuum). These matrix elements stem from nonperturbative phenomena and have to be either extracted from experiment or be determined on the lattice. In many phenomenological applications they are usually modeled in terms of various nonperturbative methods or models.

Generically, the application of QCD to hadronic processes involves the consideration of hard parton subprocesses and (unknown) nonperturbative functions to describe binding effects. Prominent examples are hard exclusive hadronic processes which involve hadron distribution amplitudes (DAs), generalized distribution amplitudes (GDAs), and generalized parton distributions (GPDs) [5, 6, 7, 8]. Applying such a framework, collisions of a real and a highly-virtual photon provide a useful tool for studying a variety of fundamental aspects of QCD.

Recently, nonperturbative quantities of a new kind were introduced—transition distribution amplitudes (TDAs) [9, 10, 11]—which are closely related to the GPDs. In contrast

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to the GDAs, the TDAs appear in the factorization procedure when the Mandelstam variable s is of the same order of magnitude as the large photon virtuality Q^2 , while t is rather small. Remarkably, there exists a reaction where both amplitude types, GDAs and TDAs, can overlap. This can happen in the fusion of a real and transversely polarized photon with a highly-virtual longitudinally polarized photon, giving rise to a final state which comprises a pair of pions. The key feature of this reaction is that it can potentially follow either path: proceed via twist-3 GDAs, or go through the leading-twist TDAs, as illustrated in Fig. 1. Such an antagonism of alternative factorization mechanisms in this reaction seems extremely interesting both theoretically and phenomenologically and deserves to be studied in detail.

The intimate relation between these two mechanisms in the production of a vector-meson pair was analyzed in [12] and it was found that these mechanisms can be selected by means of the different polarizations of the initial-state photon. In contrast, for (pseudo)scalar particles, such as the pions, this effect is absent enabling us to access the overlap region of both mechanisms and their duality as opposed to their additivity.

In this talk, we will report on the possibility for duality between these antagonistic mechanisms of factorization, associated either with GDAs or with TDAs, in the regime where *both* Mandelstam variables s and t are rather small compared to the large photon virtuality Q^2 .

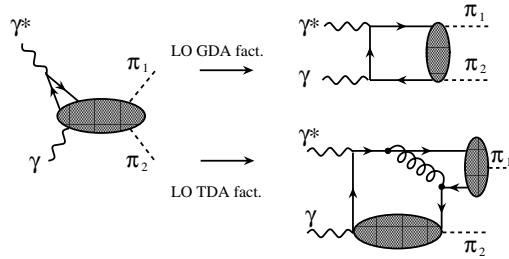


Figure 1: Two ways of factorization: via the GDA mechanism and via the TDA mechanism.

2 Regimes of Factorization within the φ_E^3 -model

Consider first the factorization of the scalar φ_E^3 model in Euclidean space. To study the four-particle amplitude in detail, it is particularly useful to employ the α -representation—see [7]. Then, the contribution of the leading “box” diagram can be written as (while details can be found in [13])

$$\mathcal{A}(s, t, m^2) = -\frac{g^4}{16\pi^2} \int_0^\infty \frac{\prod_{i=1}^4 d\alpha_i}{D^2} \exp\left[-\frac{1}{D} \left(Q^2 \alpha_1 \alpha_2 + s \alpha_2 \alpha_4 + t \alpha_1 \alpha_3 + m^2 D^2\right)\right], \quad (1)$$

where m^2 serves as a infrared (IR) regulator, $s > 0$, $t > 0$ are the Mandelstam variables in the Euclidean region, and $D = \sum_{i=1}^4 \alpha_i$. Assuming that $q^2 = Q^2$ is large compared to the mass scale m^2 (which simulates here the typical scale of soft interactions), the amplitude (1) can indeed be factorized. As regards the other two kinematic variables s and t , one can identify three distinct regimes of factorization: (a) $s \ll Q^2$ while t is of order Q^2 ; (b) $t \ll Q^2$ while s is of order Q^2 ; (c) $s, t \ll Q^2$.

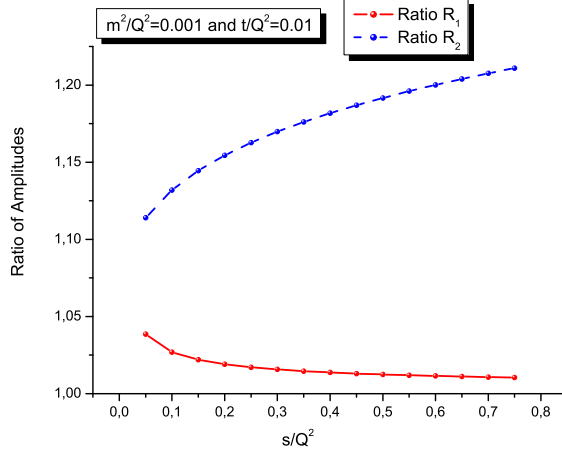


Figure 2: The ratios R_1 and R_2 as functions of s/Q^2 .

Regime (a): The process is going through the s-channel. In this regime, the main contribution in the integral in Eq. (1) arises from the integration over α_1 when $\alpha_1 \sim 0$:

$$\mathcal{A}_{\text{GDA}}^{\text{as}}(s, t, m^2) = -\frac{g^4}{16\pi^2} \int_0^\infty \frac{d\alpha_2 d\alpha_3 d\alpha_4}{D_0^2} \exp\left(-s \frac{\alpha_2 \alpha_4}{D_0} - m^2 D_0\right) \left[Q^2 \frac{\alpha_2}{D_0} + t \frac{\alpha_3}{D_0} + m^2\right]^{-1}. \quad (2)$$

Schematically this means that the propagator, parametrized by α_1 , can be associated with the partonic (hard) subprocesses, while the remaining propagator constitutes the soft part of the considered amplitude, i.e., the scalar version of the GDA.

Regime (b): Here we have to eliminate from the exponential in Eq. (1) the variables Q^2 and s , which are large. This can be achieved by integrating over the region $\alpha_2 \sim 0$. Performing similar manipulations as in regime (a), we find that the scalar TDA amplitude can be related to the scalar GDA via $\mathcal{A}_{\text{TDA}}^{\text{as}}(s, t, m^2) = \mathcal{A}_{\text{GDA}}^{\text{as}}(t, s, m^2)$.

Regime (c): The relevant regime to investigate duality is when it happens that both variables s and t are simultaneously small compared to Q^2 , i.e., when $s, t \ll Q^2$. In this case, there are two possibilities to extract the leading Q^2 -asymptotics, notably, we can either integrate over the region $\alpha_1 \sim 0$, or integrate instead over the region $\alpha_2 \sim 0$. Clearly, these two options can be associated with (i) the GDA mechanism of factorization with the meson pair scattered at a small angle in its center-of-mass system or, alternatively, (ii) with the TDA mechanism of factorization. We stress that we may face double counting when naively adding these two contributions. We interpret such a behavior as a signal of an ingrained tendency for duality between the GDA(s-channel) and the TDA (t-channel) factorization mechanisms.

In order to verify the appearance of duality we carry out a numerical investigation of the exact and the asymptotic amplitudes. In doing so, we introduce the following ratios $R_1 = \mathcal{A}_{\text{TDA}}^{\text{as}}/\mathcal{A}$ and $R_2 = \mathcal{A}_{\text{GDA}}^{\text{as}}/\mathcal{A}$. Appealing to the symmetry of these ratios under the exchange of the variables $s \leftrightarrow t$, we take t/Q^2 to be 0.01 and look for the variation of the ratios with s/Q^2 . This variation is illustrated in Fig. 2 from which one sees that in the region where s/Q^2 is rather small, i.e., in the range (0.01, 0.05), both asymptotic formulae are describing the exact amplitude with an accuracy of more than 90%. This behavior supports the conclusion that, when both Mandelstam variables s/Q^2 and t/Q^2 assume values in the wide interval (0.001, 0.7), duality between the TDA and the GDA factorization mechanisms emerges.

3 TDA- and GDA-Factorizations for $\gamma\gamma^* \rightarrow \pi\pi$

Having discussed the appearance of duality between the GDA and the TDA factorization schemes within a toy model, we now turn attention to real QCD. To analyze duality, we consider the exclusive $\pi^+\pi^-$ production in a $\gamma_T\gamma_L^*$ collision, where the virtual photon with a large virtuality Q^2 is longitudinally polarized, whereas the other one is quasi real and transversely polarized. Notice that the GDA and the TDA regimes correspond to the *same* helicity amplitudes. Given that the considered process involves a longitudinally and a transversally polarized photon, we are actually dealing with twist-3 GDAs [14]. On the other hand, for the twist-2 contribution, related to the meson DA, we use the standard parametrization of the π^+ -to-vacuum matrix element which involves a bilocal axial-vector quark operator [1]. Finally, the $\gamma \rightarrow \pi^-$ axial-vector matrix elements can be parametrized in the form, cf. [10],

$$\langle \pi^-(p_2) | \bar{\psi}(-z/2) \gamma_\alpha \gamma_5 [-z/2; z/2] \psi(z/2) | \gamma(q', \varepsilon') \rangle \stackrel{\mathcal{F}}{=} \frac{e}{f_\pi} \varepsilon'_T \cdot \Delta_T P_\alpha A_1(x, \xi, t), \quad (3)$$

where $P = (p_2 + q')/2$, and $\Delta = p_2 - q'$, and noticing that the symbol $\stackrel{\mathcal{F}}{=}$ means Fourier transformation and that the vector matrix element does not contribute here. To normalize the axial-vector TDA, A_1 , we express it in terms of the axial-vector form factor measured in the weak decay $\pi \rightarrow l\nu_l\gamma$ [15, 16, 13]. The helicity amplitude associated with the TDA mechanism reads $\mathcal{A}_{(0,j)}^{\text{TDA}} = \mathcal{F}^{\text{TDA}} \varepsilon'^{(j)} \cdot \Delta^T / |\vec{q}|$ with

$$\mathcal{F}^{\text{TDA}} = [4\pi\alpha_s(Q^2)] \frac{C_F}{2N_c} \int_0^1 dy \frac{\phi_\pi(y)}{y\bar{y}} \int_{-1}^1 dx A_1(x, \xi, t) \left(\frac{e_u}{\xi - x} - \frac{e_d}{\xi + x} \right), \quad (4)$$

where we have employed the 1-loop $\alpha_s(Q^2)$ in the $\overline{\text{MS}}$ -scheme with $\Lambda_{\text{QCD}} = 0.312$ GeV for $N_f = 3$ [17]. [Note that there is only a mild dependence on Λ_{QCD} .]

Turning now to the helicity amplitude, which includes the twist-3 GDA, we anticipate that it can be written as (see, for example, [14]) $\mathcal{A}_{(0,j)}^{\text{GDA}} = \mathcal{F}^{\text{GDA}} \varepsilon'^{(j)} \cdot \Delta^T / |\vec{q}|$ with

$$\mathcal{F}^{\text{GDA}} = 2 \frac{W^2 + Q^2}{Q^2} (e_u^2 + e_d^2) \int_0^1 dy \partial_\zeta \Phi_1(y, \zeta, W^2) \left(\frac{\ln \bar{y}}{y} - \frac{\ln y}{\bar{y}} \right), \quad (5)$$

where the partial derivative is defined by $\partial_\zeta = \partial/\partial(2\zeta - 1)$. In deriving (5), we have used for the twist-3 contribution the Wandzura-Wilczek approximation. Duality between expressions (4) and (5) may occur in that regime, where both s and t are simultaneously much smaller in comparison to the large photon virtuality Q^2 . More insight into the relative weight of the amplitudes with TDA or GDA contributions can be gained once we have modeled these non-perturbative quantities. We commence with the TDAs and, assuming a factorizing ansatz for the t -dependence of the TDAs, we write $A_1(x, \xi, t) = 2 \frac{f_\pi}{m_\pi} F_A(t) A_1(x, \xi)$, where the t -independent function $A_1(x, \xi)$ is normalized to unity. To satisfy the unity-normalization condition, we introduce a TDA defined by $A_1(x, 1) = A_1^{\text{non-norm}}(x, 1) / \int_{-1}^1 dx A_1^{\text{non-norm}}(x, 1)$ and continue with the discussion of the t -independent TDAs. Recalling that we are mainly interested in TDAs in the region $\xi = 1$ [1, 2], it is useful to adopt the following parametrization

$$A_1^{\text{non-norm}}(x, 1) = (1 - x^2) \left(1 + a_1 C_1^{(3/2)}(x) + a_2 C_2^{(3/2)}(x) + a_4 C_4^{(3/2)}(x) \right), \quad (6)$$

where a_1, a_2, a_4 are free adjustable parameters, encoding nonperturbative input, and the standard notations for Gegenbauer polynomials are used. It is not difficult to show that the

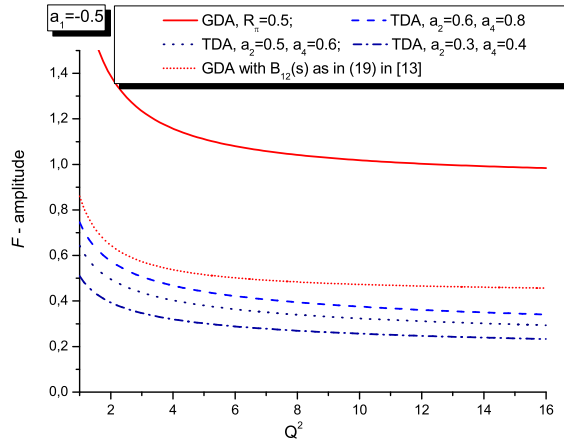


Figure 3: Helicity amplitudes \mathcal{F}^{TDA} and \mathcal{F}^{GDA} as functions of Q^2 , using $a_1 = -0.5$ found in lattice simulations. The value of s/Q^2 varies in the interval $[0.06, 0.3]$.

TDA expressed by Eq. (6) results from summing a D -term, i.e., the term with the coefficient a_1 , and meson-DA-like contributions. For our analysis, we suppose that $a_1 \equiv d_0$ [8], which is equal to -0.5 in lattice simulations. With respect to the parameters a_2 and a_4 , we allow them to vary in quite broad intervals, notably, $a_2 \in [0.3, 0.6]$ and $a_4 \in [0.4, 0.8]$, that would cover vector-meson DAs with very different profiles at a normalization scale $\mu^2 \sim 1 \text{ GeV}^2$ (see, for example, [18]). The function $\Phi_1(z, \zeta)$ is rather standard and well-known (details in [5, 13]).

We close this section by summarizing our numerical analysis of [13]. We calculated both functions \mathcal{F}^{TDA} and \mathcal{F}^{GDA} , and show the results in Fig. 3. The dashed line corresponds to the function \mathcal{F}^{TDA} , where we have adjusted the free parameters to $a_2 = 0.6$, $a_4 = 0.8$. The results, obtained for rather small values of these parameters, are displayed by the broken lines in the same figure. The dotted line denotes the function \mathcal{F}^{TDA} with $a_2 = 0.5$ and $a_4 = 0.6$, whereas the dash-dotted line employs $a_2 = 0.3$ and $a_4 = 0.4$. For comparison, we also include the results for \mathcal{F}^{GDA} . In that latter case, the dense-dotted line corresponds to the GDA amplitude, where the expression for \tilde{B}_{12} has been estimated via Eq. (20) of [13], while the solid line represents the simplest ansatz for \tilde{B}_{12} with $R_\pi = 0.5$. From this figure one may infer that when the parameter \tilde{B}_{12} , which parametrizes the GDA contribution, is estimated with the aid of the Breit-Wigner formula (provided $s, t \ll Q^2$), there is duality between the GDA and the TDA factorization mechanisms. Hence, the model for $\Phi_1(z, \zeta)$, which takes into account the corresponding resonances, can be selected by duality.

4 Conclusions

We have provided evidence that when both Mandelstam variables s and t turn out to be much less than the large momentum scale Q^2 , with the variables s/Q^2 and t/Q^2 varying in the interval $[0.001, 0.7]$, then the TDA and the GDA factorization mechanisms are equivalent to each other and operate in parallel. We have also demonstrated that duality may serve as a tool for selecting suitable models for the nonperturbative ingredients of various exclusive amplitudes entering QCD factorization. In this context, we observed that twist-3 GDAs appear to be dual to the convolutions of leading-twist TDAs and DAs, multiplied by a QCD effective coupling.

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